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Small reconstruction distance in convolution formalism

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Abstract: Different numerical wavefront propagations in Fresnel approximation are proposed in digital holography. Standard convolution formalism fails for small reconstruction distances. We developed a simplified convolution formalism that is equivalent to the angular spectrum.

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1. Introduction

Different formalisms are used in digital holographic microscopy (DHM) in order to propagate numerically a digital wavefront. The Fresnel-Kirschoff method [1], the convolution formalism 2, 3 and the angular spectrum 4. For small reconstruction distances, the Fresnel-Krischoff and the convolution method fails to reconstruct correctly the images as shown in Ref. [5]. We demonstrate in this paper that the convolution method we use is similar to the convolution formalism and allows computation of propagation with small reconstruction distances.

2. Reconstruction formalisms

In this section we present first our formulation of the convolution formalism and then we demonstrate that the

2.1 Convolution method used in our group

The convolution formulation expressed in our group is [3, 6]:

$$\Psi_{\mathrm{M}}(n,m) = \mathcal{F}^{-1}\left[\mathcal{F}\left\{RI_H\right\}\cdot G_M\right], \tag{1}$$

where I_H is the digital hologram, R the digital reference wave $\mathcal{F}$ the Fourier transform, $\mathcal{F}^{-1}$ the inverse Fourier transform, and the kernel is

$$G_M(k,l) = \exp\left[-i\pi\lambda d(\nu_k^2+\nu_l^2)\right] \tag{2}$$

$$= \exp\left[-i\frac{\pi\lambda d}{N^2\Delta x^2}(k^2+l^2)\right], \tag{3}$$

where $\nu_k = k/(N\Delta x)$, $\nu_l = l/(N\Delta y)$ are the spatial frequencies coordinates; λ the wavelength, d the reconstruction distance, N the number of pixels, Δx the pixel size and k,l entire number $-N/2 \leq k,l < N/2$.

2.1.1 Angular spectrum formulation

Kim *et al.* presents the angular spectrum method in Ref. 4, 5. The reconstructed complex wavefront is:

$$\Psi(\xi,\eta,d) = \mathcal{F}^{-1}\{filter[\mathcal{F}\{\Psi(\xi,\eta,0)\}]G_K\}, \tag{4}$$

where

$$G_K(k,l) = \exp\left[ik_z d\right] \tag{5}$$

is the kernel of the method and $filter[\mathcal{F}\{\Psi(\xi,\eta,0)\}]$ represents the filtered spectrum that keep only the frequencies associated to the virtual or the real image as already presented by Cuche *et al.* in Ref. 7. This filtering is therefore not

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associated to the angular spectrum method, but can be applied with every numerical optical propagation. The kernel $\exp[ik_z d]$ is defined by:

$$k_z(k,l) = \sqrt{k^2 - k_x^2 - k_y^2} \tag{6}$$

$$= 2\pi\sqrt{\lambda^{-2} - \left(\frac{k}{N\Delta x}\right)^2 - \left(\frac{l}{N\Delta x}\right)^2} \tag{7}$$

$$= \frac{2\pi}{\lambda}\sqrt{1 - \frac{\lambda^2}{N^2\Delta x^2}(k^2 + l^2)} \tag{8}$$

$$= \frac{2\pi}{\lambda}\sqrt{1 - x}, \tag{9}$$

where $x = \frac{\lambda^2}{N^2\Delta x^2}(m^2 + n^2)$. x is maximal when $m = n = N/2$:

$$x_{\max} = \frac{\lambda^2}{2\Delta x^2} \tag{10}$$

With $N = 512$, $\Delta x = 6.7\mu$m and $\lambda = 633$ nm, $x_{\max} = 4.46 \cdot 10^{-3}$. We could therefore developed k_z in Taylor series:

$$k_z(m,n) = \frac{2\pi}{\lambda}\left[1 - \frac{x}{2} + \frac{x^2}{8} + ...\right] \tag{11}$$

$$\cong \frac{2\pi}{\lambda} - \frac{\pi\lambda}{N^2\Delta x^2}(k^2 + l^2) \tag{12}$$

Equation 5 becomes with the second order approximation

$$G_K = \exp\left[i\frac{2\pi d}{\lambda}\right]\exp\left[-i\frac{\pi\lambda d}{N^2\Delta x^2}(k^2 + l^2)\right] \tag{13}$$

The term $\exp\left[i\frac{2\pi d}{\lambda}\right]$ is a constant phase factor that can be suppressed. We have therefore with the second order approximation:

$$G_M = G_K(2^{nd} \text{ order}) \tag{14}$$

In other words, our convolution formalism is a simplification in the second order approximation of the angular spectrum formulation.

3. Comparison

Figure 1 presents the comparison for reconstruction obtained with the convolution and angular spectrum methods of an hologram of a USAF test target magnified by a x10 MO. It appears that there are no observable difference between the angular spectrum method and the our proposed convolution for amplitude or phase images.

Figure 2(b) presents the comparison of phase profiles along the white line defined on (a). The convolution and the angular spectrum are equivalent.

4. Conclusion

We demonstrate in this paper that the convolution formalism we used is similar to the angular spectrum method. The reconstructed images are equivalent and as the angular spectrum method, small reconstruction distances can be used.

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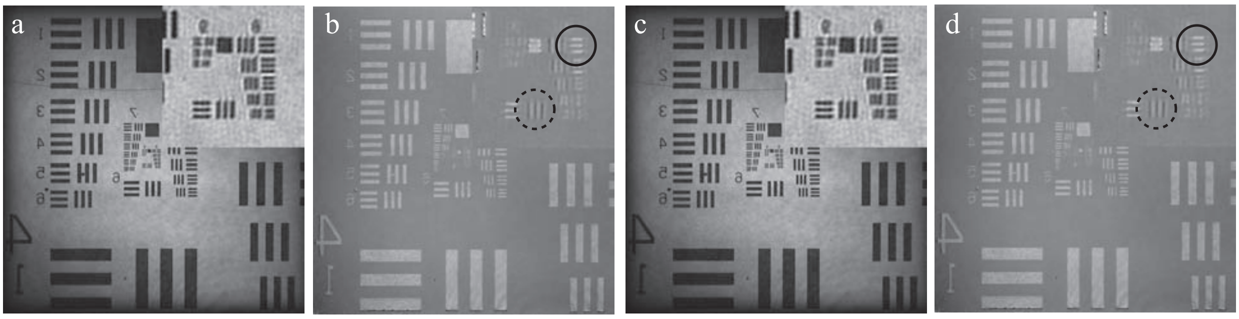

Fig. 1. Comparison for convolution (a,b) and angular spectum (c,d) methods. The amplitude (a,c) and phase (b,d) reconstruction of a USAF test target are achieved from an hologram recorded with a x10 MO.

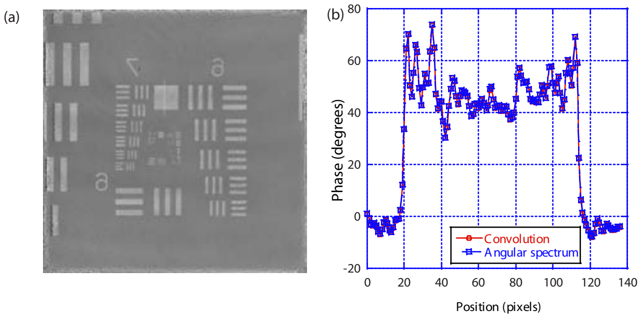

Fig. 2. (b) comparison of phase profile along white line on (a). The hologram was recorded with a x20 MO.

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