Numerical optics in digital holography

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Abstract: As digital holography achieves a fully numerical reconstruction of the complex wavefront diffracted by specimen, a numerical optics formalism has been developed: numerical filters replace pinholes in Fourier planes, numerical lenses allow aberrations compensation and image magnification.

OCIS codes: (090.1760) Computer holography; (090.1000) Aberration compensation; (110.0180) Microscopy

1. Introduction

In classical microscopy, and in optics in general, optical elements are used and optimized in order to achieve the best results. For example, lenses arrangement could be optimized for the best lateral resolution with minimum aberrations and/or distortion; pinholes are sometimes used to filter out different diffraction orders after a grating element.

On the other hand, a first application of holography is to use the holograms as optical elements to compensate for aberrations. These holograms are sometimes recorded on photographic plates [1, 2] or are generated by computer and written on spatial light modulators [3, 4].

A new branch of holography called digital holography or digital holographic microscopy showed that it is possible to recover at video frequency the complex wavefront reflected by or transmitted through a specimen [5] by using a digital camera instead of photographic plate. Most of the time, the digital hologram of a physical diffracted wavefront is recorded on a CCD camera [6] and then the digitally reconstructed wavefront is propagated digitally from the hologram plane to the image plane (see for example Refs. [7, 8]).

It was already demonstrated in several papers, that numerical procedure can be used to perform frequency filtering [9], high order aberrations and anamorphism compensation [8, 10–19] in digital holography. The purpose of this paper consists to demonstrate that numerical optics can replace physical optics. These numerical optics are also free from obstruction problems and any kind of shape for numerical lenses (NLs) are possible, allowing aberration and distortion compensation, magnification and frequency filtering.

To demonstrate the effectiveness of the numerical optics, the microscope objective of the digital holographic microscope (DHM) is replaced by a cylindrical lens to produce aberrations, distortion and different magnifications along horizontal and vertical directions. We show that numerical optics allows to reconstruct amplitude and phase images without aberration or image distortion and with the same magnification.

2. Hologram acquisition and wavefront reconstruction

The setup used is presented on Fig. 1(a): **O** is the object wave and **R** the reference wave, CL is the cylindrical lens used as microscope objective. The object is a USAF test target. The object and reference waves interfere in off-axis geometry on the CCD camera to produce the digital hologram $I_H = \mathbf{R}^2 + \mathbf{O}^2 + \mathbf{R}^*\mathbf{O} + \mathbf{R}\mathbf{O}^*$ [Fig. 1(b)]. The wavefront is reconstructed in the Fresnel approximation by using the convolution formalism [7, 8]:

$$\Psi(m,n) = NL^I(m,n) \cdot A \cdot \mathrm{FFT}^{-1}\left\{\mathrm{FFT}\left[NL^H(k,l) I_H^F(k,l)\right] \cdot \exp\left[-i\pi\lambda d(\nu_k^2 + \nu_l^2)\right]\right\}, \tag{1}$$

where, FFT is the Fast Fourier Transform; *m*, *n*, *k*, *l* are integers ($-N/2 < m,n,k,l \leq N/2$); *d*, the reconstruction distance; $A = \exp(i2\pi d/\lambda)/(i\lambda d)$; λ, the wavelength; $\nu_k = k/(N\Delta x)$, $\nu_l = l/(N\Delta y)$ are the spatial frequencies coordinates, NL^P are defined in the hologram ($P = H$) or image ($P = I$) plane. Instead of propagating the digital hologram I_H, we propagate a filtered apodized hologram

$$I_H^F = \mathrm{FFT}^{-1}[\mathrm{FFT}(I_H \cdot AP)FM], \tag{2}$$

where *AP* is a numerical amplitude filter used to apodized the hologram and therefore to suppress numerical diffraction created by the finite window of the hologram (cf. Ref. [20] for details); *FM* is a numerical mask filtering the hologram to propagate only the real (**RO***) or the virtual (**R*****O**) image. *FM* can be seen as a numerical pinhole with any shape definition [9] [the hole corresponds to the none-black pixels in Fig. 1(c)].

NL^P are complex arrays with constant amplitude ($|NL(k,l)| = 1\ \forall k,l$); the shape of the lens is defined by a standard polynomial model [18] (other models are possible, as Zernike polynomials [8]):

$$NL(k,l) = \exp\left[-i\frac{2\pi}{\lambda}\sum_{\alpha=\beta=0}^{\alpha+\beta=o} P_{\alpha\beta}\cdot k^\alpha l^\beta\right], \tag{3}$$

where $P_{\alpha\beta}$ are the NL parameters and *o* is the polynomial order. These parameters can be computed automatically by fitting the assumed to be flat areas in the reconstructed phase image with the NL model [8, 18] or by using a conjugated reference hologram [17].

Furthermore, NLs can be used for magnification. Indeed, NL can be defined by a thin lens transmittance [21]. To be able to have different magnification in horizontal (M_k) or vertical (M_l) direction, this magnification lens is written:

$$NL^{H,M}(k,l) = \exp\left[i\frac{2\pi}{\lambda}\left(\frac{k^2\Delta x^2}{2f_k}+\frac{l^2\Delta y^2}{2f_l}\right)\right]. \tag{4}$$

From Eqs. 3 and 4, the standard polynomial parameters of the numerical magnification lens ($NPL^{H,M}$) are written: $P_{20}^{H,M} = \Delta x^2/(2f_k)$ and $P_{02}^{H,M} = \Delta x^2/(2f_l)$. From lens equation and knowing the initial reconstruction distance, it is easy to determine the $NPL^{H,M}$ from the wanted magnification M_k and M_l [8].

The use of this magnification NL in the hologram plane changes the focus reconstruction distance as presented in Refs. [8, 19]. To use two different magnifications, two reconstruction distances are needed in the reconstruction propagation. This is done by changing the kernel of the propagation:

$$\exp\left[-i\pi\lambda d(\nu_k^2+\nu_l^2)\right] \longrightarrow \exp\left[-i\pi\lambda(d_k\nu_k^2+d_l\nu_l^2)\right]. \tag{5}$$

This modification was already presented in different papers in order to compensate for anamorphism and astimatism [11, 13].

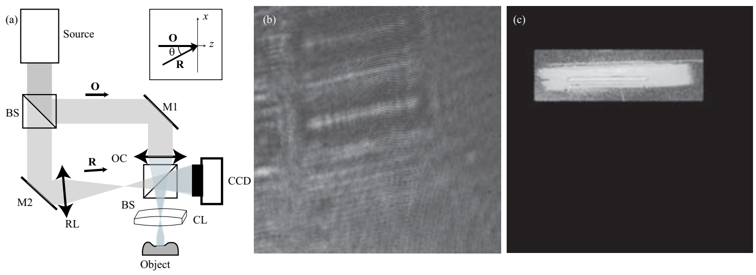

Fig. 1. (a) Digital holographic microscope: **O**, reference wave; **R**, the object wave; M1, M2, mirrors, BS, beam splitter; OC, condenser; RL, reference lens; CL, cylindrical lens. Inset presents the off-axis geometry. (b) Digital hologram recorded with a USAF test target object. (c) Filtered spectrum of (b), FM has value 0 in the dark area and 1 elsewhere.

3. Results

Figure 2 presents different reconstructions obtained from the same hologram [Fig. 1(a)] filtered with the mask FM presented in Fig. 1(c). Fig. 2(a,b) present respectively the amplitude and phase image with the parameters $P_{10}^H = -3.96426E-8$, $P_{01}^H = -1.40369E-7$, $P_{20} = P_{02} = 4.08365E-10$ and without apodization compensation ($AP = 1$). We note that the amplitude image suffers from astigmatism (horizontal edge of steps are not in focus) and the phase is disturbed by residual phase aberration. In Fig. 2(c,d), numerical apodization is applied and the astigmatism is compensated by adjusting $P_{02} = 4.22365E-10$. Furthermore the NPL in the image plane is adjusted automatically [18] to compensate for phase aberrations. In Fig. 2(e,f), we adjust $P_{11}^H = 4.1E-11$ to compensate for the distortion of the image [see insets in Figs. 2(d,e)]. Finally two different magnifications are applied in Fig. 2(g,h) ($M_l = 1,22$ and $M_k = 1.01$) to provides the correct quotient between the length and the width (5/1) given by the constructor of the USAF test target.

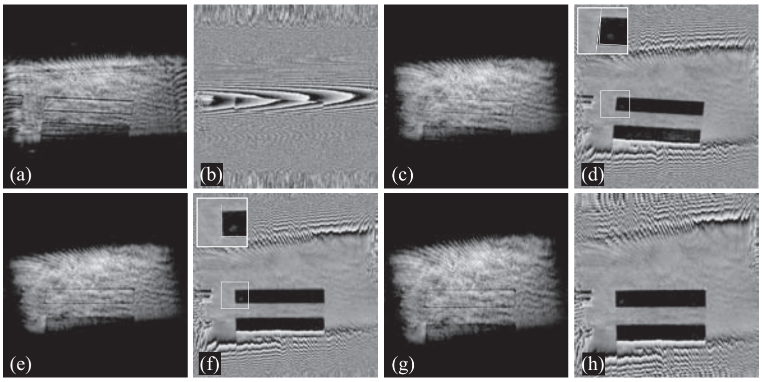

Fig. 2. Different steps of the reconstruction of amplitude and phase images of the USAF test target. (a,b) "normal" reconstruction; (b,c) apodization, astigmatism and phase aberration compensation; (c,d) distortion compensation; (e,f) magnification $M_l = 1,22$ and $M_k = 1.01$ to provide correct length and width quotient (5/1).

4. Conclusion

We demonstrate in this paper the powerful of numerical optics applied in digital holographic microscopy. Physical optics such as pinholes, apodizing optics or lenses can be replaced advantageously by numerical optics. They can have any shape and can be placed at any position avoiding obstruction problems to provide aberration- and distortion-free amplitude and phase images.

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